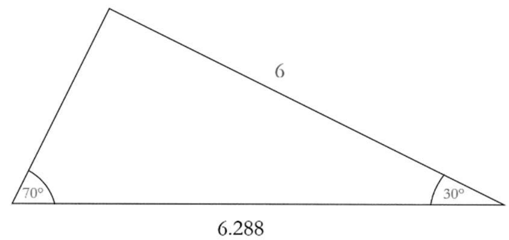
The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

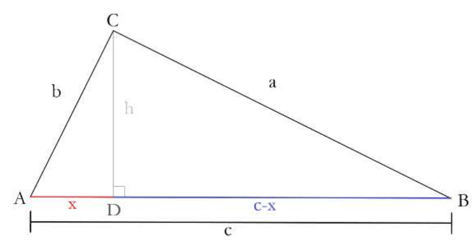
This equation can also be written $\cos A = \frac{b^2+c^2-a^2}{2bc}$, and is used to find an unknown angle or an unknown side of a triangle. The values a and A could represent any side of a triangle and angle of a triangle opposite that side respectively and $\cos A$ means the function of $\cos$ is used on the angle A.

For example, let $b = 6$ and $c = 6.288$ - meaning the remaining side can be expressed as a which is the value we want to find. The angle opposite to a is $30°$ so we shall say $A = 30°$.



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 6^2 + 6.288^2 - 2 \times 6 \times 6.288 \times \cos 30°$$
$$a^2 = 10.192 \ldots$$
$$a = \sqrt{10.192 \ldots} = 3.192 \ldots$$

Proof



Consider the triangle to the right. As we can see, ΔACD and ΔCBD are both right-angled triangles. This means we can apply Pythagoras' Theorem to both of these smaller triangles that make up the larger triangle, and later use SOH CAH TOA on one of these constituent triangles. Note that the line AB has length c, and has been split into the lines AD and DB with lengths x and $c - x$ respectively ($x + c - x = c$)

Using Pythagoras' Theorem

$$adjacent^2 + opposite^2 = \text{hypotenuse}^2$$

For ΔACD

$$x^2 + h^2 = b^2$$

Re-arranging this gives

$$h^2 = b^2 - x^2$$

For ΔCBD

$$(c - x)^2 + h^2 = a^2$$

Re-arranging this gives

$$h^2 = a^2 - (c - x)^2$$

Equating the two values for h^2

$$a^2 - (c - x)^2 = b^2 - x^2$$

Expanding $(c - x)^2$

$$a^2 - c^2 + 2cx - x^2 = b^2 - x^2$$

Re-arranging this gives

$$a^2 = b^2 + c^2 - 2cx$$

Using SOH CAH TOA

$$\cos\theta = \frac{opposite}{hypotenuse}$$

So for ΔACD

$$\cos A = \frac{x}{b}$$

Re-arranging this gives

$$x = b\cos A$$

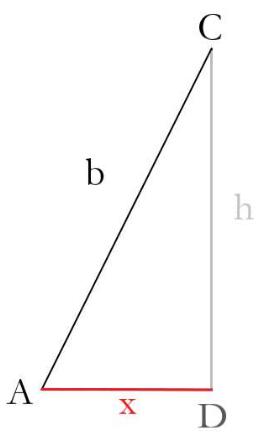
Inputting this value of x into the above equation

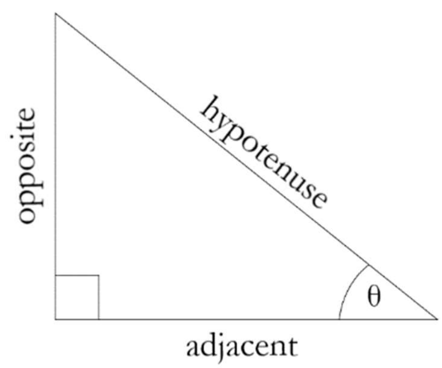
$$a^2 = b^2 + c^2 - 2cb\cos A$$

So

$$\mathrm{a}^2 = \mathrm{b}^2 + \mathrm{c}^2 - 2\mathrm{bc}\cos A$$

For reference, here is the general form for a right-angled triangle and the triangle ACD used for Pythagoras' Theorem in step 1, $\cos\theta$ in step 9 and $\cos A$ in step 10.





<u>See also</u>
- SOH CAH TOA
- Sine Area Rule
- Cosine Rule

<u>References</u>

Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. p.174